

Knowledge in Lineland

(Extended Abstract)

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ABSTRACT

In this paper we investigate a concrete epistemic situation: there are agents (humans, robots, cameras,...) and propositions (lamps on or off, obstacles dangerous or not,...) located in Lineland. We express properties with the standard epistemic logic language like “Agent A knows that agent B knows that lamp L is on”. We give some words about model-checking, satisfiability problem and common knowledge.

Categories and Subject Descriptors

I.2.4 [Theory]: Epistemic modal logic. Knowledge representation.

General Terms

Theory

Keywords

Knowledge representation. Spatial reasoning. Epistemic modal logic.

1. INTRODUCTION

In this article, we introduce a spatially grounded epistemic logic based on the simple case: a line. Our approach is different from [2] and even [4]. Here a model is directly a drawing like Figure 1 or 2 and not a Kripke model. This is motivated essentially because constructing the Kripke model by hand for a problem (e.g. Muddy Children etc.) gives the impression that we solve the problem by formalize it. With our approach, a problem is directly represented by its drawing (Figure 1).

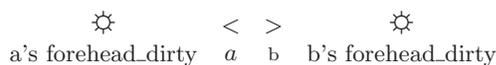


Figure 1: Muddy-children

This logic provides a pedagogic graphical model-checker for students¹ based on the same idea that [1]. On the other

¹You can find a model-checker implemented in Java/Scheme at <http://www.irit.fr/~Francois.Schwarzentruber/agentsandlamps/>.

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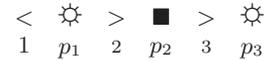


Figure 2: Example of a world

hands, it may have some application in spatial reasoning in robotics or video games like in [3].

2. SYNTAX

Our logic is based on the language of $S5_n$ [2]:

DEFINITION 1 (LANGUAGE).

Let ATM , AGT be two countable sets of respectively atomic propositions and agents. The language \mathcal{L}_{AGT} is defined by the following rule:

$$\varphi ::= \top \mid p \mid \varphi \wedge \varphi \mid \neg \varphi \mid K_a \psi$$

where $p \in ATM$ and $a \in AGT$.

As usual, $\varphi \vee \psi =^{def} \neg(\neg\varphi \wedge \neg\psi)$. $\hat{K}_a \psi =^{def} \neg K_a \neg \psi$.

The formula p is read as “the lamp p is on” and $K_a \psi$ means “agent a knows that p is true”.

3. SEMANTICS

The semantics is not defined with a class of models but geometrically. A *world* is a situation where all agents have a *location* (*position* and *direction* where they look) in the line and all *lamps* (atomic propositions) have a *position* and a *state* (on or off). Formally:

DEFINITION 2 (WORLD).

A *world* w is a tuple $\langle \leq, d, \pi \rangle$ where

- \leq is a total order over $AGT \cup ATM$;
- $d: AGT \rightarrow \{\text{left}, \text{right}\}$;
- $\pi: ATM \rightarrow \{\perp, \top\}$.

The set of all worlds is noted W . The order \leq enumerates agents and lamps from left to right. $d(a)$ denotes the direction where the agent a looks. π is a valuation.

EXAMPLE 1. *The Figure 2 gives us an example of a world $\langle \leq, d_{AGT}, \pi \rangle$. We have:*

- $1 \leq p_1 \leq 2 \leq p_2 \leq 3 \leq p_3$;
- $d(1) = \text{right}; d(2) = \text{left}; d(3) = \text{left}$;
- $\pi(p_1) = \top; \pi(p_2) = \perp; \pi(p_3) = \top$;

Now we are going to define the epistemic relation over worlds. $wR_a u$ means that agent a can not distinguish w from u , i.e. agent a sees the same things in w and u . Formally:

DEFINITION 3 (EPISTEMIC RELATION).

Let $a \in AGT$. We define the *epistemic relation* R_a on the set worlds W by $wR_a v$ iff:

- If $d(a) = \text{right}$,
 1. for all $x \in AGT \cup ATM$, ($a \leq_w x$ iff $a \leq_v x$);
 2. for all $x, y \in AGT \cup ATM$ such that $a \leq_w x$ and $a \leq_w y$, we have ($x \leq_w y$ iff $x \leq_v y$);
 3. for all $x \in AGT$, $a \leq_w x$ implies $d_w(x) = d_v(x)$;
 4. for all $x \in ATM$, $a \leq_w x$ implies $\pi_w(x) = \pi_v(x)$.
- Similarly, if $d(a) = \text{left}$ replace \leq_w by \geq_w .

Briefly, suppose that $wR_a u$ and that $d(a) = \text{right}$. In this case, $a \leq_w x$ means x is on the left of a . As $d(a) = \text{right}$ means that a is looking to the left, $a \leq_w x$ means that a sees x . The condition 1. means that agent a sees the same lamps and agents in both w and v . The condition 2. means that if two objects x or y are seen by a in w (and also in v because it equivalent from 1.) then they are in the same order both in w and v . The condition 3. means that an agent x seen by a has the same direction both in w and v . The condition 4. means that a lamp seen by agent a has the same state both in w and v . If an object x is not seen by a in w , then 1. gives it is also not seen in v but there is no more constraints over the position, direction or state of the object. Until now, we have finally defined a model $\mathcal{M} = \langle W, (R_a)_{a \in AGT}, \nu \rangle$ where ν maps each world $w \in W$ to π_w . From now, the truth conditions is standard:

DEFINITION 4 (TRUTH CONDITIONS).

Let $w \in W$. We define $w \models \varphi$ by induction:

- $w \models p$ iff $\pi(p) = \top$;
- Truth conditions for boolean connectives are standard;
- $w \models K_a \psi$ iff for all w' , $wR_a w'$ implies $w' \models \psi$.

We say that a formula φ is valid iff $\forall w \in W, w \models \varphi$.

3.1 Some validities

Since R_a is an equivalence relation on W , then the axioms T , 4 and 5 of classical epistemic logic are valid. But there are more validities in $L^{\ast 1D}$ than in $S5_n$.

The semantics of $K_a p$ in $L^{\ast 1D}$ corresponds to the fact that the agent a sees the light p and the light p is on. Informally, $K_1(p \vee q)$ means that agent 1 has a proof that $p \vee q$. In other words, either he sees p on, or he sees q on. Hence, either $K_1 p$ or $K_1 q$. More generally:

PROPOSITION 1. Let $\varphi, \psi \in \mathcal{L}_{AGT}$ such that agents and lamps appearing in φ and ψ are disjoint.

$$\models_{L^{\ast 1D}} K_1(\varphi \vee \psi) \rightarrow K_1 \varphi \vee K_1 \psi.$$

Interestingly, we have $\models_{L^{\ast 1D}} K_1 K_2 p \wedge K_2 K_1 p \rightarrow (K_1 K_2)^+ p$ where “ $(K_1 K_2)^+$ ” denotes any finite sequence of K_1 and K_2 . That is to say common knowledge comes only from $K_1 K_2 p \wedge K_2 K_1 p$ like in Figure 3.

More surprising is the fact that common knowledge is not guaranteed by $K_1 K_2 \varphi \wedge K_2 K_1 \varphi$ for all φ . Consider the world w of Figure 4. We have $w \models K_1 K_2 \neg K_2 p \wedge K_2 K_1 \neg K_2 p$. But, we have $w \not\models K_1 K_2 K_1 \neg K_2 p$.

In fact, $L^{\ast 1D}$ lacks the property of uniform substitution.

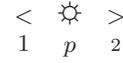


Figure 3: Common-knowledge of p

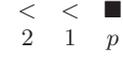


Figure 4: $w \models K_1 K_2 \neg K_2 p \wedge K_2 K_1 \neg K_2 p \wedge \neg K_1 K_2 K_1 \neg K_2 p$

4. TWO DECISION PROBLEMS

4.1 Definitions

DEFINITION 5 (MODEL-CHECKING OF $L^{\ast 1D}$).

We call *model-checking of $L^{\ast 1D}$* the following problem:

- Input: a formula $\varphi \in \mathcal{L}_{AGT}$, a world w (where only atoms and agents occurring in φ are taken in account);
- Output: Yes iff we have $w \models_{L^{\ast 1D}} \varphi$. No, otherwise.

In the Definition 5, we do not care about propositions or agents not in the formula φ . In particular, the data structure for the order \leq is a *finite* list representing a permutation over agents’ and propositions’ occurring in φ .

DEFINITION 6 ($L^{\ast 1D}$ -SATISFIABILITY PROBLEM).

We call *$L^{\ast 1D}$ -satisfiability problem* the following problem:

- Input: a formula $\varphi \in \mathcal{L}_{AGT}$;
- Output: Yes iff there exists a w s.th. $w \models_{L^{\ast 1D}} \varphi$.

PROPOSITION 2. *The model-checking of $L^{\ast 1D}$ and satisfiability problem are in PSPACE.*

Moreover, if AGT is infinite, we can reduce those two problems to Quantified propositional logic satisfiability problem and then the two problems are indeed PSPACE-complete.

5. CONCLUSION

We have presented a spatially grounded epistemic logic. One advantage is that a model is very close to the reality it represents. Furthermore, model-checking and satisfiability remains in PSPACE as for $S5_n$. From now, there are many perspectives: study in more details complexities when AGT is finite, find an axiomatization. And above all study Flatland...

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